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Combustion Dynamics and Instabilities: Elementary Coupling and Driving Mechanisms

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Elementary processes that can be involved in the development of combustion instabilities in gas turbine combustors are described. The premixed mode of combustion is considered more specifically because it is used in most advanced gas turbine systems. The processes envisaged portray the combustion dynamics of real systems, but they are analyzed in simple laboratory configurations. Among the many possible interactions, the most relevant mechanisms are those that generate fluctuations in heat release or induce pressure perturbations. Some typical paths are highlighted to help in the understanding of the multiple links that can exist between elementary processes. Processes involving acoustic/flame coupling, unsteady strain rates, flame response to inhomogeneities, interactions of flames with boundaries, and flame/vortex interactions are specifically examined. For each process, a driving or a coupling path is proposed relating heat release fluctuations to acoustic variables in certain cases or leading from acoustic variables to heat release fluctuations in other cases. Stress is also put on characteristic time lags, which are key parameters in the triggering and development of instabilities. Well-controlled experiments illustrate the many possibilities and can serve to guide the modeling effort and to validate computational tools for combustion dynamics.

I. Introduction

COMBUSTION instabilities constitute a central problem in many fields of application from aerospace propulsion systems and gas turbines operating in the premixed mode to domestic boilers and radiant heaters. Instabilities resulting from resonant interactions between driving processes and coupling modes lead to oscillations of the flow, inducing many undesirable effects: large amplitude structural vibrations, increased heat fluxes at the system walls, flashback, and flame blowoff. In some extreme cases, the outcome is a catastrophic failure.

A variety of complex physical processes may be involved in the development of instabilities, depending on the system characteristics, operating conditions, etc. Figure 1 shows some of the interactions that can lead to combustion instabilities. A large amount of experimental and theoretical work has been carried out to identify the fundamental mechanisms and to analyze the processes. There are some early observations in a classical study of Mallard and Le Châtelier.[1] An often quoted paper of Rayleigh[2] establishes a criterion stating that oscillations are sustained when heat release and pressure fluctuations are in phase. Much of the recent work has relied on detailed experimentation with advanced optical diagnostics and on numerical modeling tools. The objective has been to establish descriptive and predictive models for combustion instabilities.

Most of the instabilities observed in practical systems result from resonant interactions between combustion and coupling modes. Schematically, a driving process generates perturbations of the flow, a feedback process couples these perturbations to the driving mechanism and produces the resonant interaction that may lead to oscillations. These processes involve time lags because reactants introduced in the chamber at one instant are converted into burnt gases at a later time. Systems with delays are more readily unstable. This is easily shown by considering a second-order model featuring a linear damping (second term) and a restoring force with a delay (third term),

$$\frac{d^2x}{dt^2} + 2\zeta\omega_0\frac{dx}{dt} + \omega_0^2 x(t-\tau) = 0 \tag{1}$$

Expanding Eq. (1) in a Taylor series to first order yields

$$\frac{d^2x}{dt^2} + \omega_0(2\zeta - \omega_0\tau)\frac{dx}{dt} + \omega_0^2 x(t) = 0 \tag{2}$$

The damping coefficient is negative if $\omega_0\tau > 2\zeta$. If the delay τ is long enough with respect to the period $T = 2\pi/\omega_0$, the amplitude of any perturbation will grow exponentially. More generally, combustion instability occurs when the natural resonant time period of the flow configuration is commensurate with the characteristic time of the combustion process. The feedback process relates the downstream flow to the upstream region where the perturbations are initiated. As a consequence, acoustic wave propagation is usually responsible for the feedback path. The coupling may also involve convective modes such as entropy waves, associated with temperature fluctuations generated by the combustion process. Vorticity convected by the flow may also be part of the coupling process. When such fluctuations reach a nozzle on the downstream end of the system, they are reflected in the form of upstream propagating pressure waves.

It is important to understand the elementary processes of interaction between combustion and waves or flow perturbations (acoustics, convective modes, injection inhomogeneities, etc.), which may become driving or coupling processes under unstable conditions. No attempt will be made in what follows to describe all of the processes involved in combustion instabilities because there are already many reviews of this subject.[3–8] The choice is made to examine aspects that could typify what takes place in gas turbine combustors. This paper specifically focuses on gaseous fueled premixed systems and employs simple and well-controlled situations, which can be examined in detail, to analyze the elementary processes.

At this point, note that one additional complication is that coupling in practical systems takes place in a complex configuration and that the flow is in most cases turbulent. A large-scale effort is now being made by various groups to develop numerical tools for combustion dynamics in such structures, based in particular on large eddy simulation, for example, the computations in Refs. 9-12. This aspect is treated by Yang in this issue,[13] and it is also covered in

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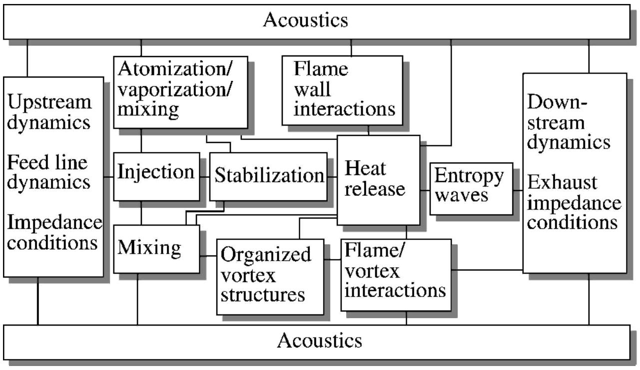


Fig. 1 Basic interactions leading to combustion instabilities (from Ref. 8).

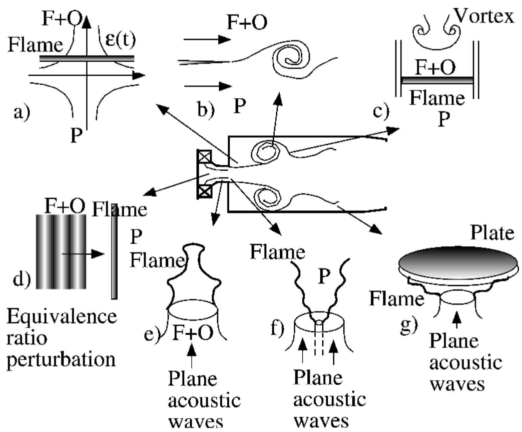


Fig. 2 Elementary processes: a) unsteady strained diffusion flame, b) unsteady strained premixed flame, c) premixed flame/vortex interaction, d) equivalence ratio perturbation interacting with a premixed flame, e) acoustically modulated conical flame, f) acoustically modulated V flame, and g) perturbed flame interacting with a plate (adapted from Ref. 8).

Refs. 7 and 8. Elementary processes such as those described subsequently should be carefully taken into account in comprehensive numerical simulation tools, and some of the experiments reviewed could clearly be taken as validation cases for these tools. Ideas developed in simple cases can be transposed to the more complex turbulent cases by noting that in many circumstances the occurrence of instability is intimately related to large-scale motion or to organized convective modes. The random turbulent fluctuations corresponding to fine-grain tubulence then act as a noisy background to the unstable oscillation. When considering the unstable process, one can focus on the organized motion, which is well illustrated in laminar experiments. Other papers in this special issue, such as those of Lieuwen[14] or Dowling,[15] provide further information on the dynamics of turbulent combustors of the type used in gas turbine systems.

Among the many possible interactions that need to be considered, some are of special relevance because they cause fluctuations in heat release or generate pressure perturbations: 1) unsteady strain rate, 2) flame/vortex interactions, 3) acoustic/flame coupling, 4) interactions of perturbed flames with boundaries, and 5) flame response to incident composition inhomogeneities. These processes are illustrated schematically in Fig. 2 and only correspond to a few blocks in Fig. 1. Many other interactions deserve attention and have already been surveyed in previous papers and in the other papers in this issue. For each of the preceding elementary processes, a driving or coupling path is proposed as an example, relating heat release to acoustic variables (pressure and velocity) in the first case and leading from acoustic variables to heat release in the other case. For ease of illustration and understanding, theses paths are associated with simple calculations or well-controlled experiments. Characteristic

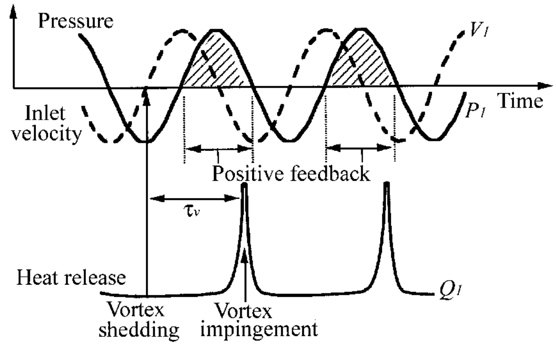


Fig. 3 Rayleigh's criterion (from Ref. 16).

time delays are introduced to show how these variables influence the development of instabilities. It is often found that convective processes determine the longest time lag in the system and are, therefore, central in the analysis of the problem (see for example, Yu et al.[16] or Smith and Zukoski[17]). This is exemplified in Fig. 3, from Ref. 16, which gives an illustration of Rayleigh's criterion for pulsating combustion modes.

It is convenient to begin with a wave equation for reacting flows (Sec. II). This relates the nonsteady heat release and the pressure field. Heat release is then considered as a pressure source in Sec. III, where two different situations are analyzed. In the first, self-sustained oscillations of a flame impinging on a plate produce an intense radiation of sound. This elementary process typifies heat release fluctuations resulting from flame-wall interactions, and one may infer from this example that similar processes may take place as a result of mutual flame interactions in the core of the flow. A second situation is that of vortex-driven fluctuations, where vortices interact with a flame, producing a heat release pulse. In Sec. IV, heat release fluctuations driven by waves or flow perturbations are dealt with. Three situations are envisaged. In the first, a conical flame is modulated by acoustic waves. It is shown that the response of the flame may be represented by a transfer function, which can be used to describe the stability map of the burner. Experimental measurements of this transfer function are compared to analytical estimates and numerical results. In the second situation, heat release perturbations result from a time-variable strain rate. A low-pass filter behavior of the flame is also found in this configuration. In the third situation, inhomogeneities formed in the upstream flow impinge on a flame producing a fluctuation in heat release. The time delay between injection and combustion is the key parameter in the process, and its defines conditions of oscillation. These three examples typify interactions that may take place in practical systems. Many other cases are treated in this issue and in the references.

II. Acoustics for Reacting Flows

Combustion instabilities can be analyzed by starting from a wave equation relating the pressure field and source terms associated with heat release and turbulence. This equation is briefly derived to highlight the relation between acoustics and combustion. More elaborate theoretical descriptions of instabilities may be developed in various other ways, as exemplified in this issue[15] or in the literature[18,19]; see Ref. 5 for a review.

A. Role of Heat Release Fluctuations

The following analysis does not provide a general framework for theoretical investigation of combustion oscillations, and its intent is much more limited. The reactive flow is assumed to be at low speed (low Mach number) because this is the case in most combustors to minimize head losses. Aerodynamic sources of sound are neglected. (A more complete description of sound sources in reactive flows may be found in Ref. 20.) The objective is to underline the role of heat release fluctuations and to demonstrate that the rate of change of these fluctuations acts as a source driving pressure waves in the system. To do this, we use many simplifications and start from the balance equations for a chemically reacting mixture of N species.[21]

One may derive a wave equation for the logarithm of the pressure (see Ref. 7),

$$\nabla \cdot \frac{c^2}{\gamma}\nabla \ln p - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\gamma}\frac{\mathrm{d}}{\mathrm{d}t}\ln p\right) = \nabla \cdot (\rho^{-1}\nabla \cdot \tau) - \frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{1}{\rho c_p T}\left[\nabla \cdot \lambda \nabla T + \Phi - \sum_{k=1}^{N} h_k \dot{w}_k - \sum_{k=1}^{N}\left(\rho Y_k c_{pk} \boldsymbol{v}_k^D \cdot \nabla T\right)\right]\right\} - \frac{\mathrm{d}^2}{\mathrm{d}t^2}(\ln R) - \nabla \boldsymbol{v} : \nabla \boldsymbol{v} \tag{3}$$

where c is the speed of sound; ρ, p, T, Y_k, $\boldsymbol{v}$, and $\boldsymbol{v}_k^D$ are the density, pressure, temperature, species mass fractions, velocity, and diffusion velocity, respectively. Φ, c_{pk}, γ, λ, and R are the viscous dissipation function, specific heats, specific heat ratio, heat conductivity, and gas constant and h_k and $\dot{w}_k$ are the specific enthalpies and rates of reaction, respectively.

It is known that in expressions similar to Eq. (3), the splitting of terms between the left- and right-hand side is somewhat arbitrary because some of the terms in the right-hand side describe features of the propagation of sound in the medium and should be included in the left-hand side. This point is discussed by Doak[22] in the context of aerodynamic sound and by Kotake[23] in a study of combustion noise. Nevertheless, it is useful to regard the terms appearing in the right-hand side of Eq. (3) as the source terms generating the pressure waves in the reactive mixture. In a turbulent reacting mixture, an order of magnitude analysis indicates that, in low-speed combustors, the dominant source terms are associated with the chemical heat release fluctuations.[23] Neglecting all other terms, one obtains

$$\nabla \cdot \frac{c^2}{\gamma}\nabla \ln p - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\gamma}\frac{\mathrm{d}}{\mathrm{d}t}\ln p\right) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\rho c_p T}\sum_{k=1}^{N} h_k \dot{w}_k\right) \tag{4}$$

If one considers low-speed reactive flows, the convective term in the material derivative may be neglected $\mathrm{d}/\mathrm{d}t \sim \partial/\partial t$. In addition when it is assumed that the specific heat ratio is constant, Eq. (4) becomes

$$\nabla \cdot c^2 \nabla \ln p - \frac{\partial^2}{\partial t^2}\ln p = \frac{\partial}{\partial t}\left(\frac{1}{\rho c_v T}\sum_{k=1}^{N} h_k \dot{w}_k\right) \tag{5}$$

This equation is not linearized, and it may be used to describe finite-amplitude waves. However, in many circumstances, the pressure waves are relatively weak, and linearization is appropriate. The pressure is then expressed as a sum of a mean and a fluctuating component: $p = p_0 + p_1$ with $p_1/p_0 \ll 1$. Then $\ln p \simeq p_1/p_0$, and Eq. (5) becomes

$$\nabla \cdot c^2 \nabla\left(\frac{p_1}{p_0}\right) - \frac{\partial^2}{\partial t^2}\left(\frac{p_1}{p_0}\right) = \frac{\partial}{\partial t}\left(\frac{1}{\rho c_v T}\sum_{k=1}^{N} h_k \dot{w}_k\right) \tag{6}$$

In practical continuous combustion devices, the mean pressure does not change by more than a few percent, the spatial derivatives of p_0 may be neglected, and, hence, Eq. (6) may be written as

$$\nabla \cdot c^2 \nabla p_1 - \frac{\partial^2}{\partial t^2}p_1 = \frac{\partial}{\partial t}\left[(\gamma - 1)\sum_{k=1}^{N} h_k \dot{w}_k\right] \tag{7}$$

In addition to Eq. (7), an expression is needed for the acoustic velocity. This may be obtained by linearizing the momentum equation and neglecting the viscous stresses. This yields

$$\frac{\partial \boldsymbol{v}_1}{\partial t} = -\frac{1}{\rho_0}\nabla p_1 \tag{8}$$

where $\boldsymbol{v}_1$ represents the velocity fluctuations. Equations (7) and (8) describe the propagation and generation of small perturbations in the reactive mixture.

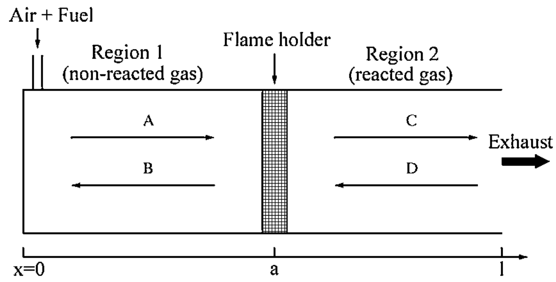


Fig. 4 Model compact flame geometry.

Considering again the source term corresponding to the nonsteady heat release, one may assume for simplicity that the chemical change occurs by a single-step reaction. Then if Δh_f° designates the change of formation enthalpy per unit mass of the mixture and if $\dot{w}$ represents the rate of reaction, the chemical source term becomes $\partial/\partial t[(\gamma - 1)(-\Delta h_f^\circ)\dot{w}]$. In most cases, the only time dependence in this expression is due to the nonsteady rate of reaction term, and as a consequence, the acoustic source term associated with the chemical reaction may be written in the form

$$(\gamma - 1)\frac{\partial Q_{1m}}{\partial t} \tag{9}$$

where Q_{1m} is the nonsteady rate of heat release per unit mass of mixture. The wave equation (7) together with the source term (9) indicate that the pressure field is driven by the nonsteady release of heat. A coupled motion can take place if this last quantity depends on acoustic variables, pressure or velocity.

B. Case of a Compact Flame in a Duct

It is instructive to briefly consider the flow of a combustible mixture through a long duct (an acoustic resonator), with a flame stabilized at the axial location $x = a$ as shown in Fig. 4 (adapted from Ref. 4). In Fig. 4, the flame zone is thin compared to the acoustic wavelength. Arrows A, B, C, and D indicate acoustic waves propagating in the system. The flame is assumed to be located at $x = a$. In this development, the following assumptions are made:

1) The frequencies of the acoustic waves considered are low compared to the duct cutoff frequency, and the perturbed motion corresponds to plane waves propagating in the axial direction.

2) The flame thickness is small compared to the acoustic wavelength so that the region of heat release may be approximated by a thin sheet located at $x = a$.

The portion of the duct upstream of the flame holder is denoted as region 1, with a fresh gas density ρ_f and sound speed c_f. Region 2 corresponds to the downstream side of the flame holder, with a burnt gas density ρ_b and sound speed c_b. The acoustic velocity v may be easily expressed in terms of upstream and downstream propagating waves. Combustion acts as a velocity source term due to the strong dilatation associated with heat release. This effect may be quantified by integrating the wave equation (7) over a thin control volume containing the flame. This leads to

$$v_b(a_+, t) - v_f(a_-, t) = (\gamma - 1)\left(Q_{1a}/\rho_f c_f^2\right) \tag{10}$$

where Q_{1a} is the instantaneous heat release rate per unit area. (See Ref. 7 for details.) When the flame is compact, the nonsteady release of heat determines the jump in acoustic velocities. Determining Q_{1a} as a function of the perturbed motion is by no means trivial. One has to relate the time-varying flow variables and the dynamic response of the flame. One approach[24] consists in using a time lag hypothesis to express Q_{1a} in terms of the time-delayed upstream velocity perturbation:

$$Q_{1a}/\rho_f c_f^2 = n v_f(a_-, t - \tau) \tag{11}$$

where n is an interaction index and τ represents a time lag. The heat release term is modeled as a function of an acoustic wave variable alone. This is clearly a simplified representation of more complex processes involving the flow, turbulence dynamics and large-scale motions, flame interactions with neighboring flames and walls, heat transfer at the boundaries, etc. Some of these processes are described later.

The value of the time lag τ relative to the frequency often defines ranges of instability. This is known from early works (for example, Ref. 19 or the review by Culick[5]), as well as from many recent studies. One possible use of an expression such as Eq. (11) is reduced modeling of active control. As shown, for example, in Ref. 4, this yields simple time lag criteria for instability development and control. In what follows, time lags will be analyzed in various laboratory-scale situations.

III. Heat Release as a Pressure Source

It is first worth examining elementary processes in which heat release acts as a pressure source term. As already mentioned, this is meant to be an illustration of more complex gas turbine combustion dynamics. The first subsection is devoted to the interaction of a flame with a wall. This mechanism may not be important for gas turbine combustors, but it serves to show that rapid changes in flame surface area can induce heat release fluctuations, which in turn may feed energy into the pressure field. This process illustrated with flame–wall interactions may also result from mutual interactions between neighboring flame elements in the core of the flow, and this may certainly arise in gas turbine combustors. Mutual interactions will be briefly analyzed in Sec. IV, dedicated to coupling processes.

A. Interactions of Flames with Boundaries

The interactions of flames with walls constitute a source of heat release fluctuations. For certain conditions, such interactions can lead to self-sustained oscillations, which are briefly described in what follows. The driving path that is considered involves surface area fluctuations, nonsteady heat release, and acoustic pressure radiation. It may be represented schematically by

$$A_1 \longrightarrow Q_1 \longrightarrow p_1$$

We will see that this allows a modeling of the instability mechanism. The analytical description relies on an understanding of the interactions between the flame and the wall, leading to an expression of the noise generated by the flame. Eventually, the determination of the time lags associated with the main mechanisms yields a complete stability map. As shown in Refs. 25–27, the impingement of a flame on a plate (simulating the presence of a wall) may constitute a possible driving process for combustion instabilities. Such experiments may also serve to show that heat release fluctuations of large amplitude can be induced by rapid changes of flame area and that these fluctuations generate an intense sound field.

In Ref. 25, a laminar premixed flame, anchored on a cylindrical burner impinges on a horizontal plate. A driver unit is used to modulate the upstream flow. Over a wide range of frequencies, the sound produced by the system is 10–20 dB higher than that emitted by a free flame submitted to the same modulation, without the plate. The interaction of the flame with the plate leads to rapid changes of the flame surface, and this constitutes a major cause of noise generation in this situation. This well-controlled experiment typifies more complicated situations in which the flame spreads in a chamber (like in a gas turbine combustor) and can produce pressure oscillations when impinging on the walls or on adjacent flame sheets. These oscillations, when correctly phased with the chamber acoustics, can lead to instabilities.

It is in fact possible to observe self-sustained oscillations of a flame impinging on a plate. The experimental setup is shown in Fig. 5. A 10-mm-thick water-cooled plate, which can move vertically, is placed above the burner. For certain distances between the plate and the burner, intense emission of sound is observed. Figure 6 shows the flame in a steady situation (Fig. 6a), when no sound emission is observed, and a complete cycle of oscillations (Figs. 6b–6e), when the instability is triggered. The flame front is undulated regularly by the perturbation, which is convected from the burner rim

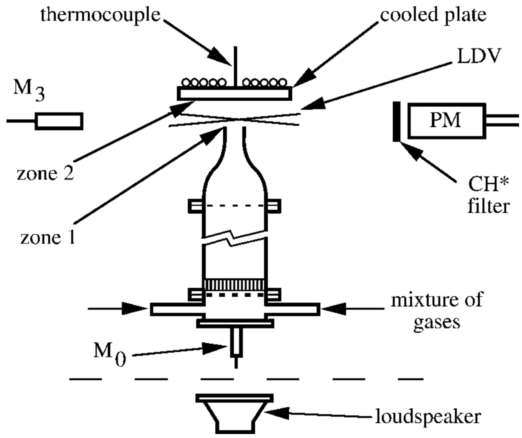


Fig. 5 Schematic of experimental setup used to study interactions of a perturbed flame with a cooled wall; configuration radiates an intense acoustic field (from Ref. 26).

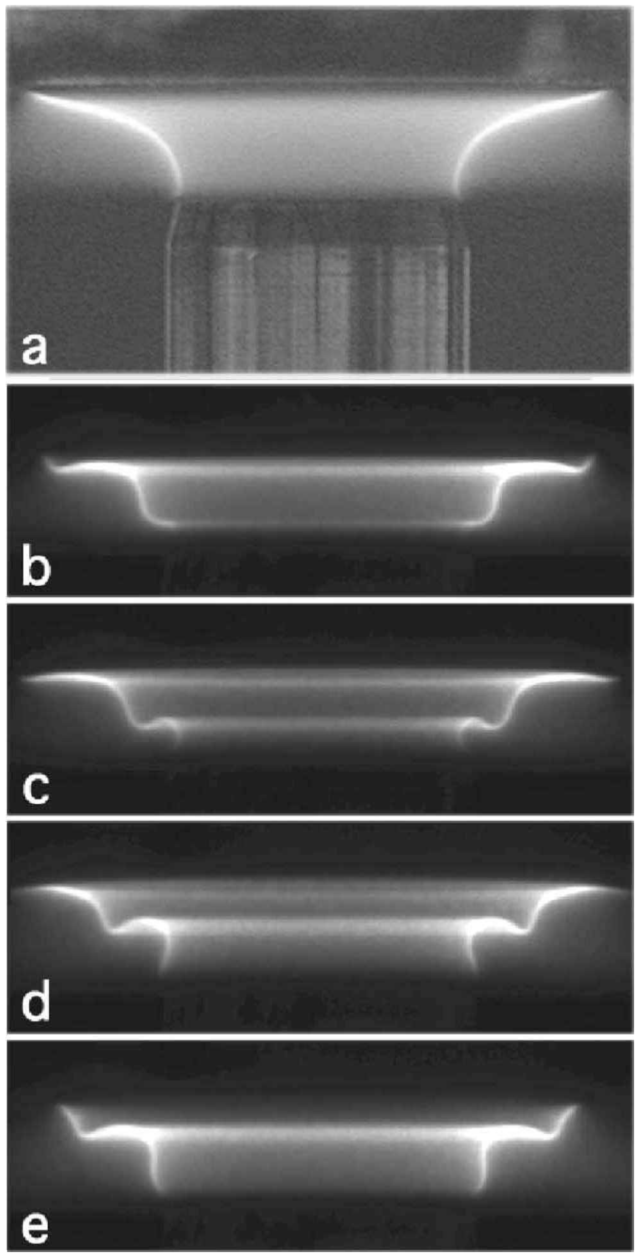


Fig. 6 Different views of the flame: a) steady state, and b–e) instantaneous images of the flame during an instability cycle (from Ref. 26).

to the plate. The sound emitted features many harmonics, with a fundamental frequency around 200 Hz.

Observations reported in Ref. 26 indicate that the burner behaves like a Helmholtz resonator with a resonance frequency of 200 Hz. The resonant behavior of the system may be described analytically by combining a model for the flame interaction with the plate and a representation of the burner acoustics. It is shown in Ref. 26 that the acoustic velocity v_1 and pressure p_1 at the burner exit may be related by a second-order equation,

$$M\frac{d^2v_1}{dt^2} + R\frac{dv_1}{dt} + kv_1 = -S_1\frac{dp_1}{dt} \quad (12)$$

where R is the system damping and k is the stiffness of the gas volume acting as a restoring force on the effective mass of air M. According to Eq. (12), the resonator is driven by external pressure fluctuations p_1 at the burner outlet. The source term on the right-hand side of Eq. (12) originates from rapid changes of the flame surface and subsequent noise radiation. Noise is generated when large portions of the flame collapse on interaction with the plate. It is known that the pressure field radiated by a compact source of nonsteady heat release takes the form[25,28–30]

$$p_\infty(r,t) = \frac{\rho_\infty}{4\pi r}\left(\frac{\rho_f}{\rho_b} - 1\right)\left[\frac{dQ}{dt}\right]_{t-\tau_a} \quad (13)$$

In this equation, ρ_∞, ρ_f, and ρ_b are the densities in the far-field air, the fresh gas, and the burned gas, respectively; and τ_a is the time required by the sound to propagate over a distance r from the sources to the detector. In the situation of gaseous premixed flames, the far-field radiated pressure p_∞ can be related to the time-retarded rate of change of the flame surface area A,

$$p_\infty(r,t) = \frac{\rho_\infty}{4\pi r}\left(\frac{\rho_f}{\rho_b} - 1\right)S_L\left[\frac{dA}{dt}\right]_{t-\tau_a} \quad (14)$$

where S_L is the laminar burning velocity. The fast rate of extinction of the flame area at the cold boundary induces a significant acoustic pressure radiation. This is shown in Fig. 7, where pressure and heat release fluctuations are essentially proportional. In Fig. 7, p_∞ is measured by a microphone. The time derivative of the heat release signal is shown at the bottom of the graph. This signal nearly coincides with the pressure signal detected by the microphone (from Ref. 26).

Fluctuations of the flame surface area A are induced by velocity perturbations at the burner exhaust. Flame perturbations, caused by velocity fluctuations v_1 at the burner outlet, are convected along the flame front toward the plate, and this can be modeled by

$$A(t) = n[v_1]_{t-\tau_c} \quad (15)$$

where n characterizes the coupling between the surface fluctuation and the velocity perturbations and τ_c is the convection time required by the perturbation to propagate from the burner exit to the plate.

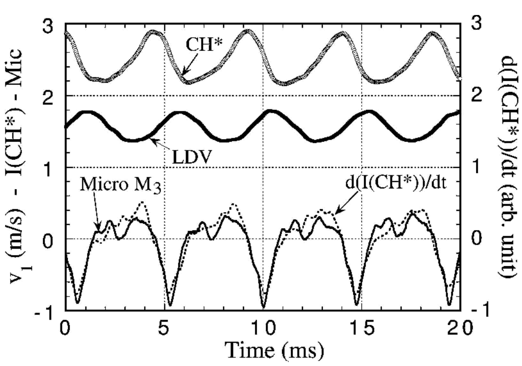


Fig. 7 Simultaneous measurements of the velocity v_1 at the burner outlet, of the CH* emission, and of the pressure p_∞ signals (from Ref. 26).

Expressions (12) and (15) may be combined, yielding a second-order equation for the velocity fluctuations,

$$\frac{d^2v_1}{dt^2} + 2\delta\frac{dv_1}{dt} + \omega_0^2 v_1 = -N\left[\frac{d^2v_1}{dt^2}\right]_{t-\tau} \quad (16)$$

where N is a normalized combustion–acoustics interaction factor and $\tau = \tau_a + \tau_c$ is a global time delay. It is shown in Ref. 26 that this model correctly retrieves the signal phase relations and reproduces the shift in frequency observed in the experiments when the burner to plate distance is varied. This confirms the existence of a mechanism whereby interactions of the flame with the wall produce high rates of surface changes, which in turn generate an intense pressure field. The stability map of the system may be determined by considering the time delays involved and the detailed balance between gain and losses in the process.

B. Flame–Vortex Interactions

Vortex structures drive various types of combustion instabilities. In many premixed systems, the ignition and delayed combustion of these structures constitute the mechanism that feeds energy into the oscillation. The process involves at least two distinct mechanisms. In the first, the flame area is rapidly changing in the presence of a vortex.[16,31] In the second, the vortex interacts with a wall, or another structure, inducing a sudden ignition of fresh material.[17]

Rollup by a vortex often controls the mixing of fresh gases into the burning regions, and this determines the nonsteady rate of conversion of reactants in the flow and the amplitude of the pressure pulse resulting from the vortex burnout. When the flame is rolled up, its surface area may increase rapidly. The growth is limited by flame shortening, resulting from interactions of neighboring elements and consumption of the reactants entrained by the vortex. Such rapid variations of flame surface correspond to the first mechanism. Flame/vortex dynamics have been studied extensively. (See Ref. 32 for a review.) Much of the experimental work has concerned toroidal or pairs of counter-rotating vortices running into a traveling premixed flame[33] or an established strained diffusion flame.[34] However, observations of combustion oscillations indicate that vortex rollup takes place while the flame develops. The vortex entrains fresh materials and hot products and ignites at a later time, producing a pulse that feeds energy in one of the resonant modes of the combustor.[35,36] This process is more difficult to study experimentally and is less well documented. Interactions between adjacent reactive vortices may also take place, leading to formation of fine-grain turbulence.

One example of self-sustained oscillations controlled by vortices is reported in Ref. 35. A multiple inlet combustor is fed with a mixture of air and propane, and it features a dump plane (Fig. 8a). The low-frequency instability observed in this case is acoustically coupled and occurs at one eigenfrequency of the system. The largest amplitude oscillations are vortex driven (Fig. 8b). The following processes are involved.

1) A vortex is shed at the dump plane when the velocity perturbation is maximum ($\boldsymbol{v}_1 \longrightarrow \Omega_1$ in a driving path, where Ω_1 represents vorticity fluctuations).

2) The vortex is convected, accelerated, and entrains hot gases from its surrounding. A combustion pulse is produced when two adjacent vortices interact, creating a large amount of small-scale turbulence and flame surface area.

3) The sudden heat release constitutes a source, which feeds energy into the perturbed acoustic motion.

This occurs when the processes are correctly phased, that is, when the time lags introduced by convection are in a suitable range with respect to the period of the motion, as shown in Fig. 9 and in Refs. 16 and 35.

Collisions of reacting vortices with boundaries are also less well covered but often observed in premixed devices.[17,37] A mechanism of this type is made evident by Kendrick et al.[37] A vortex is shed from a single inlet into a dump combustor (Fig. 10, from Ref. 38). The vortices are synchronized by one of the acoustic longitudinal modes of the system.

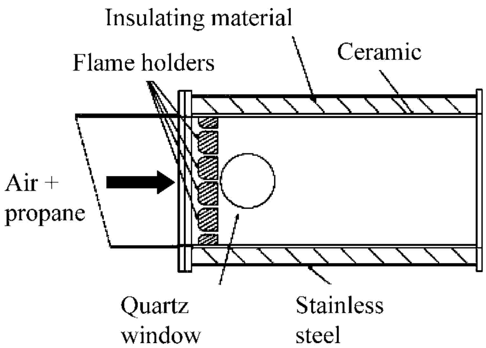


Fig. 8a Geometry of the multiple-flame-holder dump combustor studied in Ref. 35.

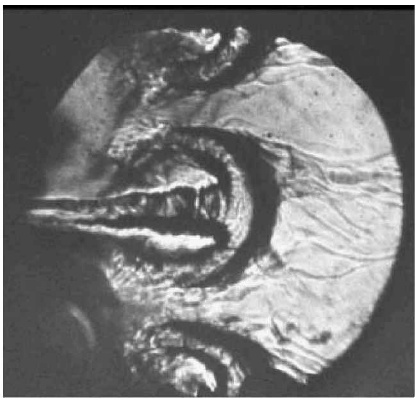


Fig. 8b Schlieren photograph of the central jet for the 530-Hz unstable regime (from Ref. 35).

Reducing the height of the combustor enhances the interaction between the vortex and the lateral boundary, producing longer axial burning regions and augmenting the overall straining of the vortex. Collision with the wall induces a rapid burning of the fresh reactants entrained by the structure. This is typical of the second mechanism invoked at the beginning of this subsection. Figure 10 shows a typical vortex-shedding event and the heat release distribution at a later instant in time.

In the first situation, the mechanism involves flow perturbations producing vorticity, which results in rapid changes of flame area, inducing a heat release pulse. In the second case, the vorticity directly causes a volumetric explosion, leading to the heat release pulse. This may globally be represented by the following expression:

$$\Omega_1 \longrightarrow Q_1 \longrightarrow p_1$$

Various other configurations have also been studied (for example, Refs. 17 and 36), and the key role of vortex structures has been underlined.

IV. Heat Release Fluctuations Driven by Waves

Unsteady fluctuations in pressure, temperature, strain rate, and induced curvature, as well as chemical composition, directly influence the rate of reaction in the flame. There is of course a direct effect of pressure, temperature, or composition on the kinetics of the system. Pressure and temperature effects are considered, for example, in Refs. 39–41, whereas in Ref. 42, the response of a distributed reaction zone to incident waves is studied. However, unsteady changes in the rates of conversion in the local flame elements or in the available flame surface area are probably more relevant. This is illustrated here by considering heat release fluctuations induced by various perturbations: Acoustic waves, unsteady strain rate, and equivalence ratio inhomogeneities are successively discussed. These are not the only possible processes by which incident perturbations drive heat release fluctuations, but they are most significant.

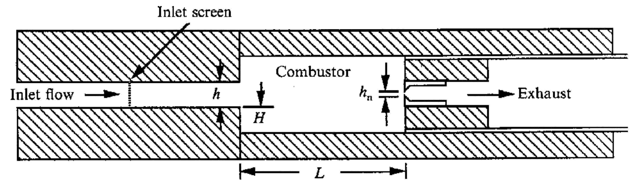


Fig. 9a Two-dimensional combustion tunnel facility studied in Ref. 16.

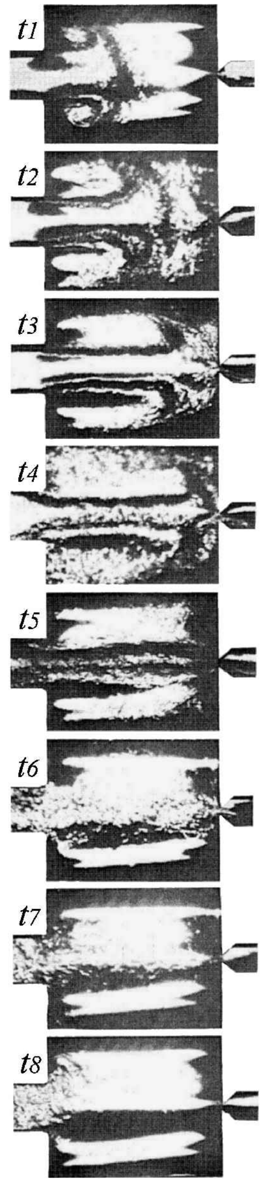


Fig. 9b Phase-locked Schlieren photographs of the combustor during an instable mode operation at the respective phases marked on the trace displayed in Fig. 9c.

A. Modulated Conical Flames

The coupling may be represented schematically by

$$p_1 \longrightarrow v_1 \longrightarrow A_1 \longrightarrow Q_1$$

This path may be investigated by modulating an initially stable flame by acoustic waves. If the geometry is simple enough, one may determine the flame response to incident perturbations. There are early investigations[43–45] of this type. If the process remains in the linear regime, one may define a transfer function between the incident velocity fluctuations and the nonsteady heat release. This will depend on the burner geometry, operating parameters, and steady-state flame configuration.

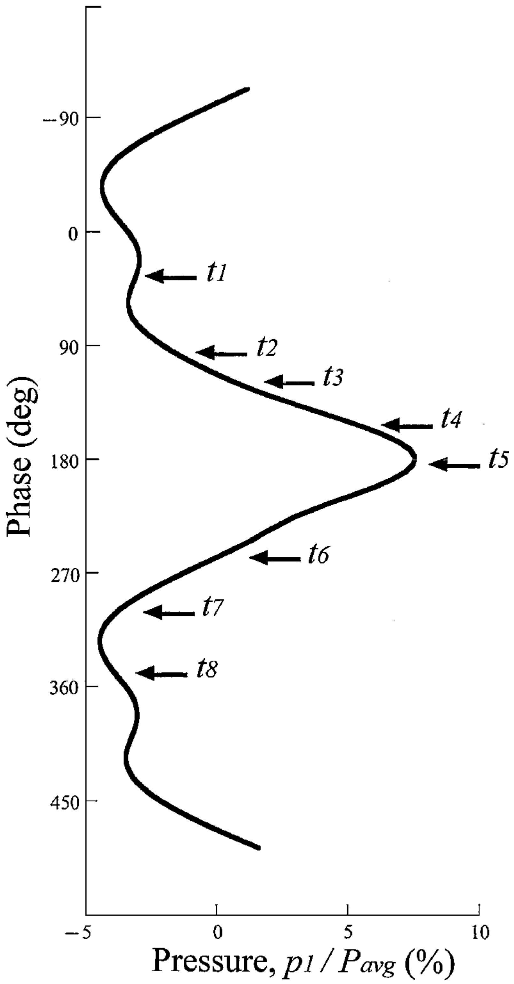


Fig. 9c Trace pressure record of one instability cycle (from Ref. 16).

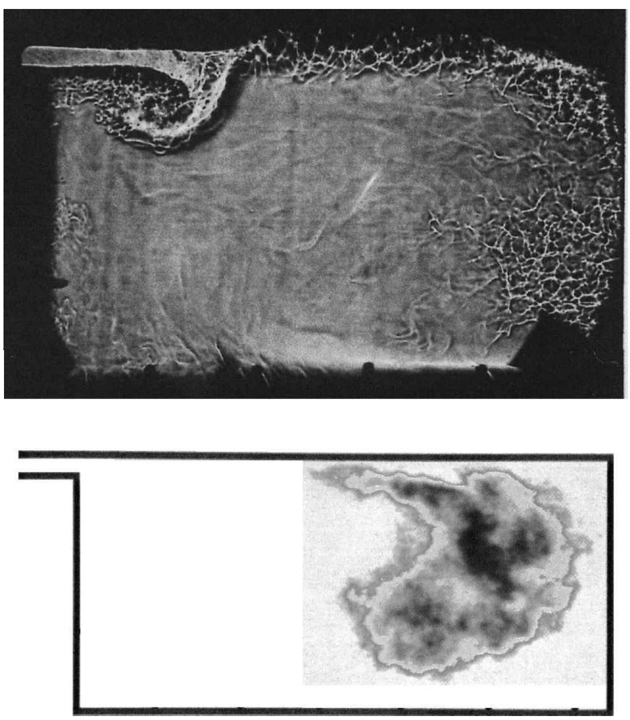


Fig. 10 Schlieren photograph and chemiluminescence image representing the heat release rate distribution at a later time during the instability cycle (from Ref. 38).

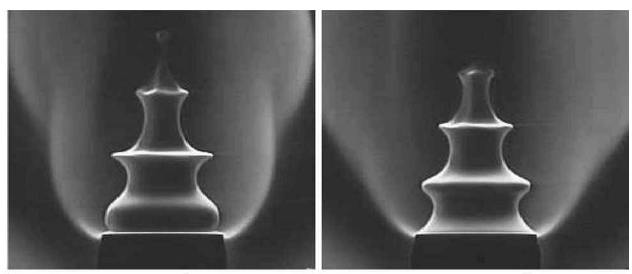
Transfer functions proposed in Refs. 46–49 indicated that the flame behaves like a low-pass filter, providing a qualitative representation of the flame response. Laminar conical flames are considered by Blackshear,[44] De Sœte,[45] and more recently Baillot et al.[50] Further theoretical efforts by Fleifil et al.[51] and a combination of theoretical analysis and detailed measurements by Ducruix et al.[52] have advanced the understanding of the problem. More recent work by Schuller et al.[53] provides additional clues on flame response in the high-frequency range. A similar approach is used by Dowling[54] to derive a model for the low-frequency nonlinear response of a ducted V flame in a geometry close to that considered in an earlier work by Marble and Candel.[55]

The modulation of a conical flame is now considered in further detail. The objective is to describe the unsteady rate of heat release as a function of acoustic variables. A laminar premixed flame is anchored on a cylindrical burner, and it is submitted to acoustic waves generated by a loudspeaker placed at the bottom of the burner. The flame response is driven by the acoustic velocity, and the aim is to find the transfer function between heat release fluctuations and velocity modulations,

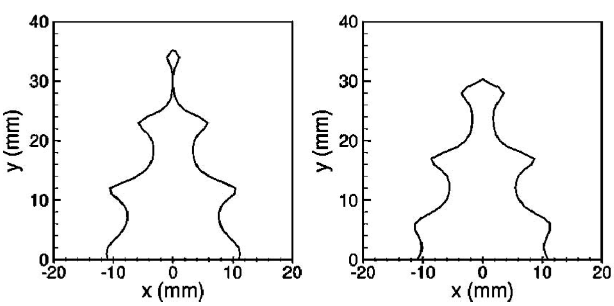
$$F(\omega) = \frac{Q_1(\omega)/Q_0}{v_1(\omega)/v_0} \tag{17}$$

where ω is the angular frequency of the modulation. The modulus of F gives the amplitude of heat release fluctuations as a function of velocity modulations, whereas its phase characterizes the time lag existing between velocity and heat release fluctuations. A complete analysis of this problem can be found in Refs. 52 and 56. Selected results are highlighted next.

The burner consists of a converging nozzle, which is water cooled, and a cylindrical tube 120 mm long, placed upstream from the nozzle and containing various grids and honeycombs to produce a laminar flow. The conical flame is stabilized on a 22-mm-diam burner rim. A loudspeaker is placed at the base of the burner. Perturbations wrinkle the flame front, and the shape of the perturbed flame depends on the frequency and the amplitude of modulation. The typical flame shapes shown in Fig. 11 are visualized with instantaneous four-color Schlieren. The use of modern diagnostic techniques has provided novel information concerning the geometry of the flame front,[52] the velocity field at the burner exhaust and in the flowfield,[56] and the



a)



b)

Fig. 11 Methane air flame modulated by acoustic perturbations; f_e = 150.5 Hz, $\omega_* \simeq 28$, $\bar{v}$ = 1.44 ms^{-1}, $v'/\bar{v}$ = 0.13, and Φ = 1.05: a) schlieren images for two different instants and b) corresponding numerical simulations.

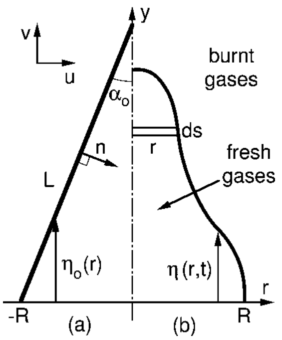


Fig. 12 Geometry of the flame in a) steady situation and b) perturbed case (from Ref. 52).

local and global heat release rates. This has allowed direct measurements of the flame transfer function defined by Eq. (17). These measurements can be compared to theoretical and numerical predictions.

An analytical transfer function can be derived by decomposing the flow in mean and perturbed components. The geometry of the problem is shown in Fig. 12. A G-equation is used to describe the flame position,

$$\frac{\partial G}{\partial t} + \boldsymbol{v} \cdot \nabla G = -S_D |\nabla G| \tag{18}$$

where $\boldsymbol{v} = (u, v)$ is the velocity vector and S_D is the flame displacement speed. In what follows, S_D is assumed to be a constant and equal to the laminar burning velocity S_L. The G variable increases from the fresh mixture to the burnt gases, and one contour $G = G_0$ represents the flame. In the simplest velocity perturbation model, the radial component u_1 is supposed to be negligible compared to v_1. The vertical component is assumed to be uniform and sinusoidal: $v_1 = v_1 \cos \omega t$. This corresponds to a bulk motion of the fresh stream. Figure 12 shows that G may be replaced by $\eta - y$, where η designates the flame position. Substituting $\eta = \eta_0 + \eta_1$ in Eq. (18), where η_0 represents the steady flame shape and $\eta_1 \ll \eta_0$ represents a perturbation (Fig. 12b), one may expand the resulting equation to first order and obtain

$$\frac{\partial \eta_1}{\partial t} = S_L \cos \alpha_0 \frac{\partial \eta_1}{\partial r} + v_1 \tag{19}$$

where α_0 denotes the half-angle of the steady flame cone.

The heat release fluctuations may be evaluated from the flame surface variations,

$$A_1 = 2\pi \cos \alpha_0 \int_0^R \eta_1 \, \mathrm{d}r \tag{20}$$

Heat release fluctuations Q_1 are directly related to the area fluctuations of flame surface: $Q_1 = \rho_f S_L q A_1$, where ρ_f is the unburnt gas density and q is the heat release per unit mass of mixture. Some calculations yield the following expression for the relative heat release fluctuations:

$$Q_1/Q_0 = (v_1/v_0)\left(2/\omega_*^2\right)[(1 - \cos \omega_*) \cos(\omega t) + (\omega_* - \sin \omega_*) \sin(\omega t)] \tag{21}$$

and the transfer function is easily deduced therefrom. The resulting expression depends on a reduced frequency $\omega_* = \omega R/(S_L \cos \alpha_0)$, where R is the burner radius. Formulation (21) may be used as a source term in Eq. (7), allowing a complete simulation of a system featuring an initially conical flame.

The analytical flame response to acoustic modulations obtained in this way relies on many simplifying assumptions. It was assumed that the perturbed velocity is axial and uniform. It is shown in Ref. 56 that this is only valid in the low-frequency range ($\omega_* < 2$). Data obtained with particle image velocimetry[56] show that the earlier model assumptions may be acceptable for slightly wrinkled flames, with a small radial component of the velocity field, that is, in the low-frequency range. In this case, the flame responds as if it

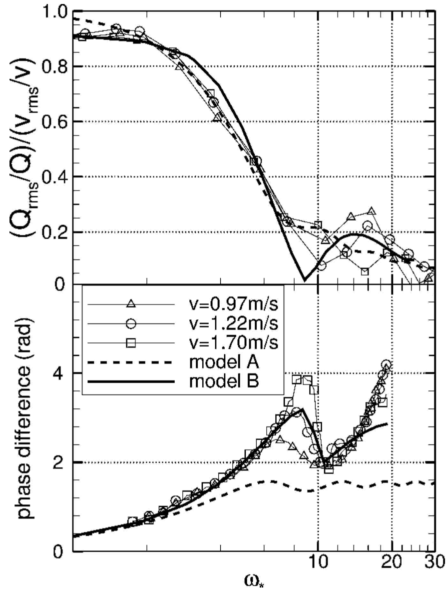


Fig. 13 Transfer functions of conical flame, model A (——) with axial, uniform perturbation velocity and model B (– – –) with perturbation velocity convected in axial direction and symbols: measurements (from Ref. 56).

were globally stretched and compressed by the modulation, while keeping an essentially conical shape. In contrast, these assumptions are too strong for larger frequencies to represent correctly the acoustic/flame interactions. In that range, velocity fields in the fresh stream exhibit important gradients, and a radial flow exists near the burner exhaust.

An alternative model represents the convective nature of the perturbed motion and the related phase differences. This is developed in Ref. 56, where a revised formulation of the velocity modulation incident on the flame is proposed. This formulation is combined with the G-equation (18), and a level-set approach is then used in the numerical integration of this equation.[56] Typical results of calculations shown in Fig. 11 are very close to the experimental flame shapes. The experimental and bulk velocity model A transfer function amplitudes essentially agree, but this is not the case for the phase (Fig. 13). The experimental phase increases with frequency, whereas the theoretical phase corresponding to expression (21) tends to $\pi/2$. When the convective model B for the velocity perturbation is used, the results are notably improved.

It is also possible to derive a new expression for the transfer function by making use of an earlier analysis of perturbed oblique flames.[57] It is shown in Ref. 53 that this function depends on two parameters, ω_* and $S_L/\bar{v}$, and one obtains an improved agreement with the experimental data. The phase of the transfer function shifts from a purely convective behavior for elongated flames to a saturated value for flat flames.[53]

B. Unsteady Strain Rate Effects

An unsteady field strain rate can be induced by the resonant acoustic motion acting on the flow. This field may change the rate of heat release in two major ways. The first consists of perturbations in the flame surface area. To analyze this first possibility, let us consider a model equation for the flame surface density:

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}t} = \epsilon \Sigma - \beta \Sigma^2 \tag{22}$$

This model is extensively used in turbulent combustion. In this expression, the first term on the right-hand side represents production of surface density by strain rate, and the second term describes mutual annihilation of flame surface density (flame shortening). At equilibrium, $\mathrm{d}\Sigma_0/\mathrm{d}t = 0$ and $\epsilon_0 \Sigma_0 - \beta \Sigma_0^2 = 0$. Let us now consider a sinusoidal perturbation of the strain rate: $\epsilon = \epsilon_0 + \epsilon_1 \cos \omega t$. This produces a perturbation in surface density: $\Sigma = \Sigma_0 + \Sigma_1$. Injecting this expression in the balance equation (22) and only retaining first-order terms in perturbations, one obtains

$$\frac{d\Sigma_1}{dt} + \epsilon_0 \Sigma_1 = (\epsilon_1 \cos\omega t)\Sigma_0 \tag{23}$$

Clearly, this will act as a low-pass filter. The steady-state solution takes the general form:

$$\Sigma_1/\Sigma_0 = \left[\epsilon_1 \big/ \left(\epsilon_0^2 + \omega^2\right)\right](\epsilon_0 \cos\omega t + \omega \sin\omega t) \tag{24}$$

In the low-frequency limit, $\omega \ll \epsilon_0$, the relative perturbation of flame surface density is in phase with the strain rate,

$$\Sigma_1/\Sigma_0 = (\epsilon_1/\epsilon_0)\cos\omega t \tag{25}$$

In the high-frequency limit, $\omega \gg \epsilon_0$, the relative perturbation of flame surface density is in quadrature with the strain rate,

$$\Sigma_1/\Sigma_0 = (\epsilon_1/\omega)\sin\omega t \tag{26}$$

and the amplitude decreases with the frequency. This type of interaction modulates the flame surface density and can be represented schematically by

$$p_1 \longrightarrow v_1 \longrightarrow \text{flow} \longrightarrow \epsilon_1 \longrightarrow A_1$$

This mechanism applies equally well to premixed and nonpremixed flames. The second type of interaction involves a direct effect on the reaction rate per unit flame surface. This is represented by

$$p_1 \longrightarrow v_1 \longrightarrow \text{flow} \longrightarrow \epsilon_1 \longrightarrow \dot{\omega}_1$$

This second effect is more important in the nonpremixed case. The reaction rate is directly related to the species gradients at the flame, which are fixed by the strain rate. In the premixed case, the consumption rate is less influenced by the strain rate, except near extinction conditions.

The flame response to strain rate has been studied quite extensively in turbulent combustion.[7,58,59] Other studies deal with the response of flames to external strain rate modulations. The problem is envisaged experimentally[60] and often treated by direct calculations using time-dependent solutions of strained flames with complex chemistry.[61,62] Analytical expressions of the flame response have also been worked out using asymptotics.[63] It is found that flames behave like low-pass filters when the perturbed strain rate fluctuations do not exceed the extinction value. When nonpremixed flames are considered and an infinitely fast chemistry limit is assumed, it is shown in Ref. 8 that the flame transfer function defined in the frequency domain as the ratio of the relative reaction rate modulation to the relative strain rate perturbation,

$$F(\omega) = \left[\frac{\tilde{\dot{m}}(\omega) - \dot{m}_0}{\dot{m}_0}\right] \Big/ \left[\frac{\tilde{\epsilon}(\omega) - \epsilon_0}{\epsilon_0}\right] \tag{27}$$

takes the form of a low-pass filter,

$$F(\omega) = \tfrac{1}{2}\{1/[1 + i(\omega/2\epsilon_0)]\} \tag{28}$$

The effect of unsteady strain on premixed flames cannot be described in such simple terms. Numerical calculations[64] indicate that the reponse of the flame to modulated strain rates takes the form of cycles around the steady-state line. The size of the cycle diminishes as the frequency increases.

It is known that flame surface area is augmented when the strain rate acting on the reactive elements is lower than the extinction value. Conversely, the flame area is limited by a mechanism of mutual interactions of adjacent reactive elements. This has been identified as a fundamental process reducing the flame surface area, for example, see Refs. 65 and 66. Mutual flame interaction is illustrated in Fig. 14. This mechanism may also influence the dynamics of turbulent flames. The rapid consumption of reactants trapped between

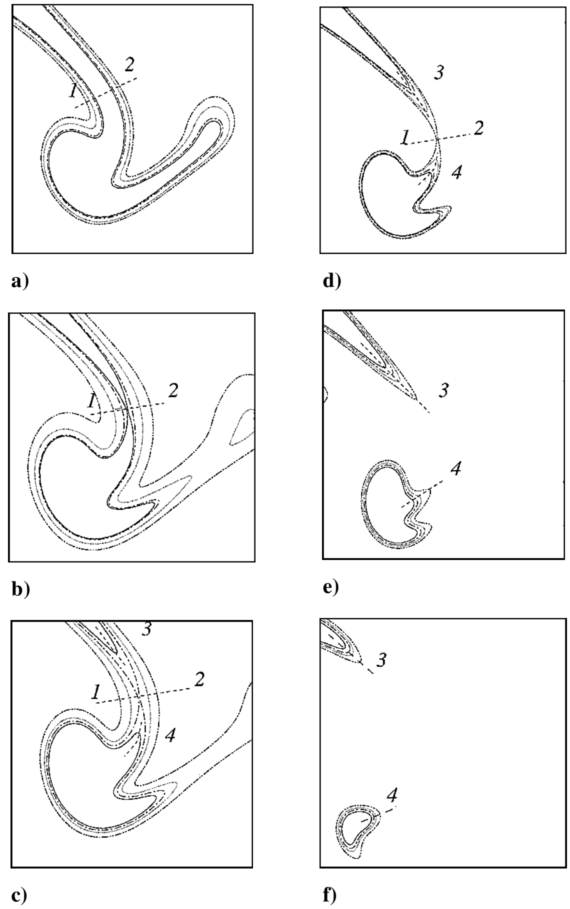


Fig. 14 Direct numerical simutation computations of the mutual flame annihilation as a limitation mechanism for flame surface production, peak consumption rates of CH_4, O_2, H_2 and CO at a) $0.61t_f$, b) $0.72t_f$, c) $0.75t_f$, d) $0.78t_f$, e) $0.81t_f$, f) $1.1t_f$, where t_f is the flame time (from Ref. 66).

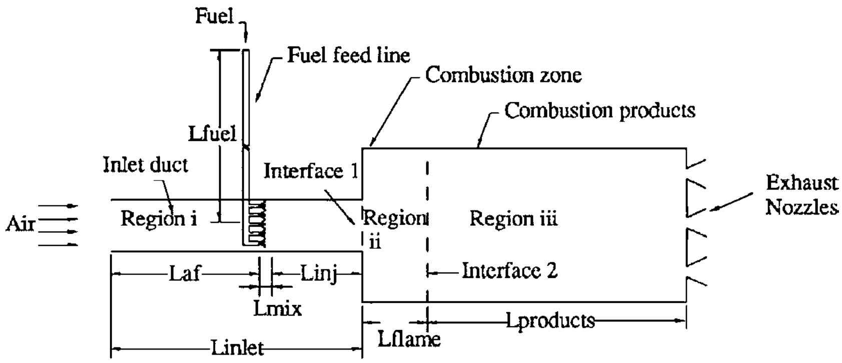
two adjacent flames may also produce a heat release pulse and the subsequent emission of pressure waves.[67] If this interaction is properly phased with respect to an acoustic eigenmode, it may drive the unstable motion. The mutual interaction of strained flames is now well understood in cases where the flame elements tend to propagate away from each other. The case of strained elements approaching each other and leading to a shortening of the flame is less easy to study experimentally. The possibility of having synchronized interactions leading to instability is not generally considered. This type of motion has been observed experimentally at least in Ref. 25.

C. Flame Response to Composition Inhomogeneities

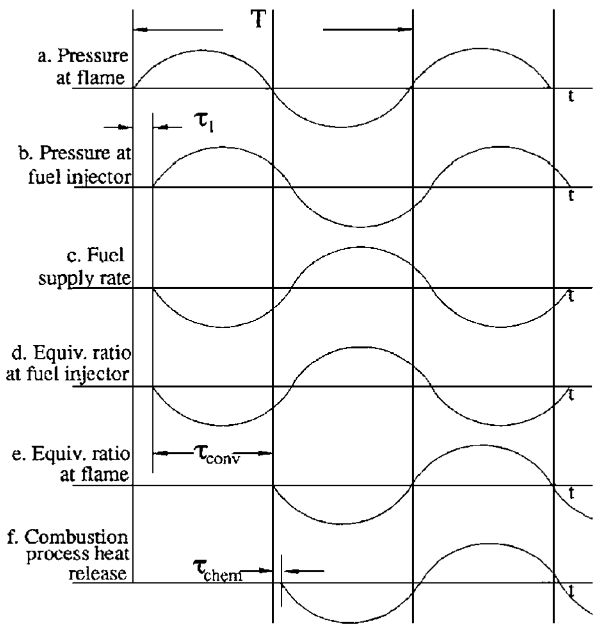
Experiments and theoretical analysis indicate that certain types of instabilities in lean premixed combustors may be driven by perturbations in the fuel and air ratio.[68–71] This is illustrated here by assuming that pressure oscillations in the combustor interact with the fuel supply line and change the fuel flow rate. A positive pressure excursion produces a decrease of the fuel supply at a later instant. This causes a negative perturbation in the equivalence ratio ϕ_1, which is then convected by the flow to the flame zone. The interaction may also take place with the air supply, and this will also affect the equivalence ratio. The two types of interactions will produce a heat release perturbation, which if properly phased with the pressure may feed energy in the resonant acoustic mode involved in the process. This interaction can be represented schematically by

$$p_1 \longrightarrow \phi_1 \longrightarrow \text{convection by the flow} \longrightarrow Q_1$$

This mechanism is represented schematically in Fig. 15. In a first step, a pressure oscillation arises in the system. This will modify the flow rate of fuel and change the equivalence ratio. Three time



a)



b)

Fig. 15 Instability driven by equivalence ratio perturbations: a) combustor model and b) time traces of pressures, equivalence ratios, and heat release in the flame (from Ref. 68).

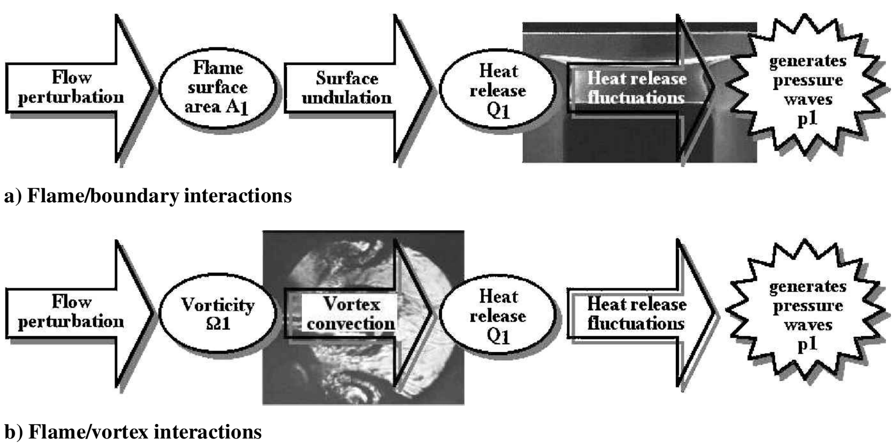
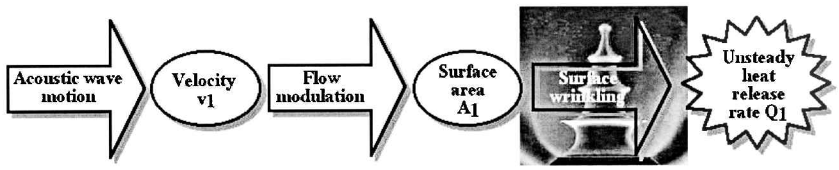


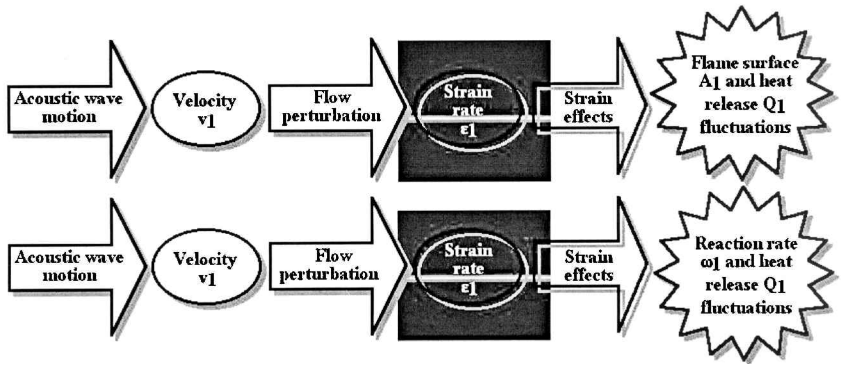
Fig. 16 Summary of driving processes.

delays define the process. The first τ_i corresponds to a phase shift between the pressure at the injector and fuel mass flow rate $\dot{m}_{F1}$. Oscillations in this flow rate induce fluctuations in the equivalence ratio ϕ_1. An inhomogeneous mixture is then convected to the reaction zone with a delay τ_{conv}. The response of the flame to the impinging fluctuations ϕ_1 follows after a combustion delay τ_{chem}. Oscillations will be sustained by this process if the pressure and heat release fluctuations are in phase. This is the case if the total delay is such that $\tau_i + \tau_{\text{conv}} + \tau_{\text{chem}} = (2n-1)T/2$, where T is the period. In many cases, the dominant delay is that associated with convection, and the last condition becomes $\tau_{\text{conv}} \simeq (2n-1)T/2$.

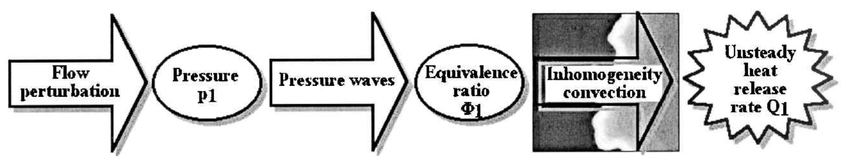
One fundamental aspect of this process is the response of the flame to incoming equivalence ratio perturbations.[72] Another aspect that will also influence this mechanism is the intensity of mixing taking place between the injector and the flame. If this mixing is efficient, the initial level of fluctuations will be diminished to a great extent,



a) Flame response to upstream modulations



b) Flame response to strain rate



c) Flame response to composition inhomogeneities

Fig. 17 Summary of coupling processes; when interacting with the proper phase lag, driving and coupling mechanisms can lead to combustion instabilities.

reducing the fluctuation in heat release. Effects of inhomogeneities are also examined in more detail in other papers in this issue.[14]

Finally Figs. 16 and 17 summarize the different paths examined in Sec. III and IV.

V. Conclusions

The development of predictive methods for combustion instabilities is an important technological objective. This prediction is now essential to the development of advanced combustors for gas turbines. Considerable progress has been made in this direction. Experiments and detailed analysis have generated a wealth of information on the basic processes involved. The present review illustrates some of these processes and focuses on the driving and coupling relations that exist between heat release fluctuations and the pressure field. With the use of well-controlled experiments, it is shown that rapid changes of the flame surface generate an intense radiation of sound. In practical situations, there are many possible mechanisms that may produce or destroy flame surface at a fast rate such as flame/wall interactions and collisions between adjacent flames or between neighboring flow structures such as vortices or reactant jets. These processes may feed energy into a resonant mode if they are properly phased with respect to the pressure. Fast changes in flame surface area constitute an important driving process of combustion instabilities.

The coupling between the pressure field and the combustion process may take many different forms. It is illustrated here with a set of experiments with laminar flames, but previous experiments on turbulent ducted configurations have indicated that premixed flames are quite susceptible to such modulations.[3] The flame is highly wrinkled by the external field, giving rise to surface and heat release fluctuations. In simple cases, it is possible to define a transfer function between the relative velocity and heat release fluctuations. Comparisons between analytical models, numerical simulations, and experiments are reviewed. It is shown that simple filter models do not provide a suitable description of the phase when the modulation frequency is high and that more refined methods must be used to get a better description of this quantity. In practical systems, the flame may also be modulated by many other means. Flame modulation may result from the field of variable strain rate, which can be induced by the nonsteady motion in the combustor. The variable strain rate can produce or diminish the flame surface area and modify the local rate of reaction per unit surface. If the fluctuations are suitably phased, they will feed energy back into the acoustic motion. Equivalence ratio perturbations caused by the differential response of the injection system may also induce heat release fluctuations when these perturbations convected downstream reach the flame. This has been identified as a possible driving process for some types of gas turbine instabilities.

Although information accumulated over many years of research is quite substantial, additional fundamental experiments and intermediate-scale investigations are still needed. Further modeling, with a focus on coupling and driving processes, is required together with detailed simulations. The many results gathered in the recent past could be used to check numerical tools and validate combustion dynamics simulations. The interactions examined in this paper only portray some of the mechanisms involved in the more complex dynamics of gas turbine combustors, a subject covered in further detail in this special issue.

Acknowledgments

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